Properization: constructing proper scoring rules via Bayes acts

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Abstract

Scoring rules serve to quantify predictive performance. A scoring rule is proper if truth telling is an optimal strategy in expectation. Subject to customary regularity conditions, every scoring rule can be made proper, by applying a special case of the Bayes act construction studied by Grünwald and Dawid (Ann Stat 32:1367–1433, 2004) and Dawid (Ann Inst Stat Math 59:77–93, 2007), to which we refer as properization. We discuss examples from the recent literature and apply the construction to create new types, and reinterpret existing forms, of proper scoring rules and consistent scoring functions. In an abstract setting, we formulate sufficient conditions under which Bayes acts exist and scoring rules can be made proper.

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