## Spatially homogeneous copulas

## Fabrizio Durante<sup>1</sup> · Juan Fernández Sánchez<sup>2</sup> · Wolfgang Trutschnig<sup>3</sup>

Received: 24 July 2017 / Revised: 1 November 2018 / Published online: 1 December 2018 © The Institute of Statistical Mathematics, Tokyo 2018

## Abstract

We consider spatially homogeneous copulas, i.e. copulas whose corresponding measure is invariant under a special transformations of  $[0, 1]^2$ , and we study their main properties with a view to possible use in stochastic models. Specifically, we express any spatially homogeneous copula in terms of a probability measure on [0, 1) via the Markov kernel representation. Moreover, we prove some symmetry properties and demonstrate how spatially homogeneous copulas can be used in order to construct copulas with surprisingly singular properties. Finally, a generalization of spatially homogeneous copulas to the so-called (m, n)-spatially homogeneous copulas is studied and a characterization of this new family of copulas in terms of the Markov \*-product is established.

Keywords Copulas · Dependence · Probability measures · Singular measures

Fabrizio Durante fabrizio.durante@unisalento.it

> Juan Fernández Sánchez juanfernandez@ual.es

> Wolfgang Trutschnig wolfgang@trutschnig.net

- <sup>2</sup> Grupo de Investigación de Análisis Matemático, Universidad de Almería, Carretera Sacramento s/n, La Cañada de San Urbano, 04120 Almería, Spain
- <sup>3</sup> Department for Mathematics, University of Salzburg, Hellbrunner Strasse 34, 5020 Salzburg, Austria

CrossMark

<sup>&</sup>lt;sup>1</sup> Dipartimento di Scienze dell'Economia, Università del Salento, Campus Ecotekne - Palazzina C, Via Monteroni 165, 73100 Lecce, Italy