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Semiparametric *M*-estimation with non-smooth criterion functions

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Abstract

We are interested in the estimation of a parameter θ that maximizes a certain criterion function depending on an unknown, possibly infinite-dimensional nuisance parameter h. A common estimation procedure consists in maximizing the corresponding empirical criterion, in which the nuisance parameter is replaced by a nonparametric estimator. In the literature, this research topic, commonly referred to as semiparametric M-estimation, has received a lot of attention in the case where the criterion function satisfies certain smoothness properties. In certain applications, these smoothness conditions are, however, not satisfied. The aim of this paper is therefore to extend the existing theory on semiparametric M-estimators, in order to cover non-smooth M-estimators as well. In particular, we develop 'high-level' conditions under which the proposed M-estimator is consistent and has an asymptotic limit. We also check these conditions for a specific example of a semiparametric M-estimator coming from the area of classification with missing data.

Keywords Asymptotic distribution · Classification · Empirical processes · M-Estimation · Missing data · Nonstandard asymptotics · Nuisance parameter · Semiparametric regression

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