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Convergence rates for kernel regression in infinite-dimensional spaces

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Abstract

We consider a nonparametric regression setup, where the covariate is a random element in a complete separable metric space, and the parameter of interest associated with the conditional distribution of the response lies in a separable Banach space. We derive the optimum convergence rate for the kernel estimate of the parameter in this setup. The small ball probability in the covariate space plays a critical role in determining the asymptotic variance of kernel estimates. Unlike the case of finite-dimensional covariates, we show that the asymptotic orders of the bias and the variance of the estimate achieving the optimum convergence rate may be different for infinite-dimensional covariates. Also, the bandwidth, which balances the bias and the variance, may lead to an estimate with suboptimal mean square error for infinite-dimensional covariates. We describe a data-driven adaptive choice of the bandwidth and derive the asymptotic behavior of the adaptive estimate.

Keywords Adaptive estimate · Bias-variance decomposition · Gaussian process · Maximum likelihood regression · Mean square error · Optimal bandwidth · Small ball probability · t process

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