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On the limiting distribution of sample central moments

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Abstract

We investigate the limiting behavior of sample central moments, examining the special cases where the limiting (as the sample size tends to infinity) distribution is degenerate. Parent (non-degenerate) distributions with this property are called *singular*, and we show in this article that the singular distributions contain at most three supporting points. Moreover, using the *delta*-method, we show that the (second-order) limiting distribution of sample central moments from a singular distribution is either a multiple, or a difference of two multiples of independent Chi-square random variables with one degree of freedom. Finally, we present a new characterization of normality through the asymptotic independence of the sample mean and all sample central moments.

Keywords Sample central moments · Singular distributions · Second-order approximation · Characterization of normality · *Delta*-method

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