




# Inference on a distribution function from ranked set samples

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## Abstract

Consider independent observations  $(X_i, R_i)$  with random or fixed ranks  $R_i$ , while conditional on  $R_i$ , the random variable  $X_i$  has the same distribution as the  $R_i$ -th order statistic within a random sample of size  $k$  from an unknown distribution function  $F$ . Such observation schemes are well known from ranked set sampling and judgment post-stratification. Within a general, not necessarily balanced setting we derive and compare the asymptotic distributions of three different estimators of the distribution function  $F$ : a stratified estimator, a nonparametric maximum-likelihood estimator and a moment-based estimator. Our functional central limit theorems generalize and refine previous asymptotic analyses. In addition, we discuss briefly pointwise and simultaneous confidence intervals for the distribution function with guaranteed coverage probability for finite sample sizes. The methods are illustrated with a real data example, and the potential impact of imperfect rankings is investigated in a small simulation experiment.

**Keywords** Conditional inference · Confidence band · Empirical process · Functional limit theorem · Moment equations · Imperfect ranking · Relative asymptotic efficiency · Unbalanced samples

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