



# Estimating quantiles in imperfect simulation models using conditional density estimation

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## Abstract

In this article, we consider the problem of estimating quantiles related to the outcome of experiments with a technical system given the distribution of the input together with an (imperfect) simulation model of the technical system and (few) data points from the technical system. The distribution of the outcome of the technical system is estimated in a regression model, where the distribution of the residuals is estimated on the basis of a conditional density estimate. It is shown how Monte Carlo can be used to estimate quantiles of the outcome of the technical system on the basis of the above estimates, and the rate of convergence of the quantile estimate is analyzed. Under suitable assumptions, it is shown that this rate of convergence is faster than the rate of convergence of standard estimates which ignore either the (imperfect) simulation model or the data from the technical system; hence, it is crucial to combine both kinds of information. The results are illustrated by applying the estimates to simulated and real data.

**Keywords** Conditional density estimation · Quantile estimation · Imperfect models ·  $L_1$  error · Surrogate models · Uncertainty quantification

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