

Maximum likelihood estimation of autoregressive models with a near unit root and Cauchy errors

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Abstract This paper studies maximum likelihood estimation of autoregressive models of order 1 with a near unit root and Cauchy errors. Autoregressive models with an intercept and with an intercept and a linear time trend are also considered. The maximum likelihood estimator (MLE) for the autoregressive coefficient is $n^{3/2}$ -consistent with n denoting the sample size and has a mixture-normal distribution in the limit. The MLE for the scale parameter of Cauchy distribution is $n^{1/2}$ -consistent, and its limiting distribution is normal. The MLEs of the intercept and the linear time trend are $n^{1/2}$ - and $n^{3/2}$ -consistent, respectively. It is also shown that the t statistic for the null hypothesis of a unit root based on the MLE has a standard normal distribution in the limit. In addition, finite-sample properties of the MLE are compared with those of the least square estimator (LSE). It is found that the MLE is more efficient than the LSE when the errors have a Cauchy distribution or a distribution which is a mixture of Cauchy and normal distributions. It is also shown that empirical power of the MLE-based t test for a unit root is much higher than that of the Dickey–Fuller t test.

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Keywords Autoregressive model · Near unit root · Cauchy distribution · Maximum likelihood estimator · Infinite variance