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# Simultaneous confidence bands for the distribution function of a finite population in stratified sampling

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**Abstract** Stratified sampling is one of the most important survey sampling approaches and is widely used in practice. In this paper, we consider the estimation of the distribution function of a finite population in stratified sampling by the empirical distribution function (EDF) and kernel distribution estimator (KDE), respectively. Under general conditions, the rescaled estimation error processes are shown to converge to a weighted sum of transformed Brownian bridges. Moreover, simultaneous confidence bands (SCBs) are constructed for the population distribution function based on EDF and KDE. Simulation experiments and illustrative data example show that the coverage frequencies of the proposed SCBs under the optimal and proportional allocations are close to the nominal confidence levels.

**Keywords** Confidence band · Stratified population distribution · Allocation · Brownian bridge · Kernel · Superpopulation

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