

Limiting distributions of likelihood ratio test for independence of components for high-dimensional normal vectors

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Abstract Consider a p -variate normal random vector. We are interested in the limiting distributions of likelihood ratio test (LRT) statistics for testing the independence of its grouped components based on a random sample of size n . In classical multivariate analysis, the dimension p is fixed or relatively small, and the limiting distribution of the LRT is a chi-square distribution. When p goes to infinity, the chi-square approximation to the classical LRT statistic may be invalid. In this paper, we prove that the LRT statistic converges to a normal distribution under quite general conditions when p goes to infinity. We propose an adjusted test statistic which has a chi-square limit in general. Our comparison study indicates that the adjusted test statistic outperforms among the three approximations in terms of sizes. We also report some numerical results to compare the performance of our approaches and other methods in the literature.

Keywords Likelihood ratio test · Covariance matrix · Independence · High-dimensional normal vector · Central limit theorem · Chi-square approximation

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