

# Asymptotic properties of the realized skewness and related statistics

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**Abstract** The recent empirical works have pointed out that the realized skewness, which is the sample skewness of intraday high-frequency returns of a financial asset, serves as forecasting future returns in the cross section. Theoretically, the realized skewness is interpreted as the sample skewness of returns of a discretely observed semimartingale in a fixed interval. The aim of this paper is to investigate the asymptotic property of the realized skewness in such a framework. We also develop an estimation theory for the limiting characteristic of the realized skewness in a situation where measurement errors are present and sampling times are stochastic.

**Keywords** High-frequency data · Itô semimartingale · Jumps · Microstructure noise · Realized skewness · Stochastic sampling

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