

## M-based simultaneous inference for the mean function of functional data

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**Abstract** Estimating and constructing a simultaneous confidence band for the mean function in the presence of outliers is an important problem in the framework of functional data analysis. In this paper, we propose a robust estimator and a robust simultaneous confidence band for the mean function of functional data using M-estimation and B-splines. The robust simultaneous confidence band is also extended to the difference of mean functions of two populations. Further, the asymptotic properties of the M-based mean function estimator, such as the asymptotic consistency and asymptotic normality, are studied. The performance of the proposed robust methods and their robust-ness are demonstrated with an extensive simulation study and two real data examples.

Keywords Confidence band  $\cdot$  Functional data analysis  $\cdot$  Robust statistics  $\cdot$  Spline smoothing  $\cdot$  M-estimator  $\cdot$  Pseudo-data

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