

Wishart exponential families on cones related to tridiagonal matrices

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Abstract Let G be the graph corresponding to the graphical model of nearest neighbor interaction in a Gaussian character. We study Natural Exponential Families (NEF) of Wishart distributions on convex cones Q_G and P_G , where P_G is the cone of tridiagonal positive definite real symmetric matrices, and Q_G is the dual cone of P_G . The Wishart NEF that we construct include Wishart distributions considered earlier for models based on decomposable(chordal) graphs. Our approach is, however, different and allows us to study the basic objects of Wishart NEF on the cones Q_G and P_G . We determine Riesz measures generating Wishart exponential families on Q_G and P_G , and we give the quadratic construction of these Riesz measures and exponential families. The mean, inverse-mean, covariance and variance functions, as well as moments of higher order, are studied and their explicit formulas are given.

Keywords Wishart distribution · Graphical model · Nearest neighbor interaction

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