

# A generalized urn with multiple drawing and random addition

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**Abstract** In this paper, we consider an unbalanced urn model with multiple drawing. At each discrete time step  $n$ , we draw  $m$  balls at random from an urn containing white and blue balls. The replacement of the balls follows either opposite or self-reinforcement rule. Under the opposite reinforcement rule, we use the stochastic approximation algorithm to obtain a strong law of large numbers and a central limit theorem for  $W_n$ : the number of white balls after  $n$  draws. Under the self-reinforcement rule, we prove that, after suitable normalization, the number of white balls  $W_n$  converges almost surely to a random variable  $W_\infty$  which has an absolutely continuous distribution.

**Keywords** Unbalanced urn · Stochastic approximation · Martingale · Maximal inequality

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