

## Waiting time for consecutive repetitions of a pattern and related distributions

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**Abstract** Let *k* be a positive integer. Some exact distributions of the waiting time random variables for *k* consecutive repetitions of a pattern are derived in a sequence of independent identically distributed trials. It is proved that the number of equations of conditional probability generating functions for deriving the distribution can be reduced to less than or equal to the length of the basic pattern to be repeated consecutively. By using the result, various properties of the distributions of usual runs are extended to those of consecutive repetitions of a pattern. These results include some properties of the geometric distribution of order *k* and those of the waiting time distributions of the ( $k_1$ ,  $k_2$ )-events. Further, the probability generating function of the number of non-overlapping occurrences of *k* consecutive repetitions of a pattern can be written in an explicit form with *k* as a parameter. Some recurrence relations, which are useful for evaluating the probability mass functions, are also given.

**Keywords** Geometric distribution of order  $k \cdot \text{Repetition of a pattern} \cdot \text{Waiting time}$  for a pattern  $\cdot$  Conditional probability generating function  $\cdot (k_1, k_2)$ -Event

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