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An asymptotic expansion for the normalizing constant of the Conway–Maxwell–Poisson distribution

Robert E. Gaunt[1] · **Satish Iyengar**[2] · **Adri B. Olde Daalhuis**[3] · **Burcin Simsek**[2]

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Abstract The Conway–Maxwell–Poisson distribution is a two-parameter generalization of the Poisson distribution that can be used to model data that are under- or over-dispersed relative to the Poisson distribution. The normalizing constant $Z(\lambda, \nu)$ is given by an infinite series that in general has no closed form, although several papers have derived approximations for this sum. In this work, we start by using probabilistic argument to obtain the leading term in the asymptotic expansion of $Z(\lambda, \nu)$ in the limit $\lambda \to \infty$ that holds for all $\nu > 0$. We then use an integral representation to obtain the entire asymptotic series and give explicit formulas for the first eight coefficients. We apply this asymptotic series to obtain approximations for the mean, variance, cumulants, skewness, excess kurtosis and raw moments of CMP random variables. Numerical results confirm that these correction terms yield more accurate estimates than those obtained using just the leading-order term.

Keywords Conway–Maxwell–Poisson distribution · Normalizing constant · Approximation · Asymptotic series · Generalized hypergeometric function · Stein's method

✉ Robert E. Gaunt
robert.gaunt@manchester.ac.uk

[1] School of Mathematics, The University of Manchester, Manchester M13 9PL, UK

[2] Department of Statistics, University of Pittsburgh, 1800 Wesley W. Posvar Hall, 230 S Bouquet Street, Pittsburgh, PA 15260, USA

[3] Maxwell Institute and School of Mathematics, The University of Edinburgh, James Clerk Maxwell Building, The King's Buildings, Peter Guthrie Tait Road, Edinburgh EH9 3FD, UK