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Pseudo-Gibbs sampler for discrete conditional distributions

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Abstract Conditionally specified models offers a higher level of flexibility than the joint approach. Regression switching in multiple imputation is a typical example. However, reasonable-seeming conditional models are generally not coherent with one another. Gibbs sampler based on incompatible conditionals is called pseudo-Gibbs sampler, whose properties are mostly unknown. This article investigates the richness and commonalities among their stationary distributions. We show that Gibbs sampler replaces the conditional distributions iteratively, but keep the marginal distributions invariant. In the process, it minimizes the Kullback–Leibler divergence. Next, we prove that systematic pseudo-Gibbs projections converge for every scan order, and the stationary distributions share marginal distributions in a circularly fashion. Therefore, regardless of compatibility, univariate consistency is guaranteed when the orders of imputation are circularly related. Moreover, a conditional model and its pseudo-Gibbs distributions have equal number of parameters. Study of pseudo-Gibbs sampler provides a fresh perspective for understanding the original Gibbs sampler.

Keywords Incompatibility · Iterative conditional replacement · Kullback–Leibler information divergence · Multiple imputation · Scan order · Stationary distribution

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