

Sobolev-Hermite versus Sobolev nonparametric density estimation on $\ensuremath{\mathbb{R}}$

Denis Belomestny 1,2 \cdot Fabienne Comte 3 \cdot Valentine Genon-Catalot 3

Received: 27 September 2016 / Revised: 11 April 2017 / Published online: 23 October 2017 © The Institute of Statistical Mathematics, Tokyo 2017

Abstract In this paper, our aim is to revisit the nonparametric estimation of a square integrable density f on \mathbb{R} , by using projection estimators on a Hermite basis. These estimators are studied from the point of view of their mean integrated squared error on \mathbb{R} . A model selection method is described and proved to perform an automatic bias variance compromise. Then, we present another collection of estimators, of deconvolution type, for which we define another model selection strategy. Although the minimax asymptotic rates of these two types of estimators are mainly equivalent, the complexity of the Hermite estimators is usually much lower than the complexity of their deconvolution (or kernel) counterparts. These results are illustrated through a small simulation study.

Fabienne Comte fabienne.comte@parisdescartes.fr

> Denis Belomestny denis.belomestny@uni-due.de

Valentine Genon-Catalot valentine.genon-catalot@parisdescartes.fr

- ¹ Faculty of Mathematics, Duisburg-Essen University, Thea-Leymann-Str. 9, 45127 Essen, Germany
- ² National Research University Higher School of Economics, Shabolovka, 26, 119049 Moscow, Russia
- ³ MAP5 UMR CNRS 8145, Université Paris Descartes, Sorbonne Paris Cité, 45 rue des Saints-Pères, 75 270 Paris Cedex 06, France

D.B. acknowledges the financial support from the Russian Academic Excellence Project "5-100" and from the Deutsche Forschungsgemeinschaft (DFG) through the SFB 823 "Statistical modeling of nonlinear dynamic processes".

Keywords Complexity \cdot Density estimation \cdot Hermite basis \cdot Model selection \cdot Projection estimator