

# Multiscale inference for a multivariate density with applications to X-ray astronomy

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**Abstract** In this paper, we propose methods for inference of the geometric features of a multivariate density. Our approach uses multiscale tests for the monotonicity of the density at arbitrary points in arbitrary directions. In particular, a significance test for a mode at a specific point is constructed. Moreover, we develop multiscale methods for identifying regions of monotonicity and a general procedure for detecting the modes of a multivariate density. It is shown that the latter method localizes the modes with an effectively optimal rate. The theoretical results are illustrated by means of a simulation study and a data example. The new method is applied to and motivated by the determination and verification of the position of high-energy sources from X-ray observations by the Swift satellite which is important for a multiwavelength analysis of objects such as Active Galactic Nuclei.

**Keywords** Multiple tests · Modes · Multivariate density · X-ray astronomy

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