

# The continuous-time triangular Pólya process

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Received: 21 April 2016 / Revised: 24 October 2016 / Published online: 2 January 2017  
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**Abstract** We study poissonized triangular (reducible) urns on two colors, which we take to be white and blue. We analyze the number of white and blue balls after a certain period of time has elapsed. We show that for balanced processes in this class, a different scaling is needed for each color to produce nontrivial limits, contrary to the distributions in the usual irreducible urns which only require the same scaling for both colors. The limit distributions (of the scaled variables) underlying triangular urns are Gamma. The technique we use couples partial differential equations with the method of moments applied in a bootstrapped manner to produce exact and asymptotic moments. For the dominant color, we get exact moments, while relaxing the balance condition. The exact moments include alternating signs and Stirling numbers of the second kind.

**Keywords** Urn · Pólya urn · Pólya process · Branching process · Gamma distribution · Partial differential equation

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