

## Additional aspects of the generalized linear-fractional branching process

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Received: 21 October 2015 / Revised: 8 June 2016 / Published online: 19 July 2016 © The Institute of Statistical Mathematics, Tokyo 2016

Abstract We derive some additional results on the Bienyamé–Galton–Watsonbranching process with  $\theta$ -linear fractional branching mechanism, as studied by Sagitov and Lindo (Branching Processes and Their Applications. Lecture Notes in Statistics— Proceedings, 2016). This includes the explicit expression of the limit laws in both the subcritical cases and the supercritical cases with finite mean, and the long-run behavior of the population size in the critical case, limits laws in the supercritical cases with infinite mean when the  $\theta$  process is either regular or explosive, and results regarding the time to absorption, an expression of the probability law of the  $\theta$ -branching mechanism involving Bell polynomials, and the explicit computation of the stochastic transition matrix of the  $\theta$  process, together with its powers.

**Keywords** Bienyamé–Galton–Watson-branching process  $\cdot \theta$ -linear-fractionalbranching mechanism  $\cdot$  Population growth  $\cdot$  Yaglom limits  $\cdot$  Powers of probability transition matrix

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