

## Statistical estimation of composite risk functionals and risk optimization problems

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**Abstract** We address the statistical estimation of composite functionals which may be nonlinear in the probability measure. Our study is motivated by the need to estimate coherent measures of risk, which become increasingly popular in finance, insurance, and other areas associated with optimization under uncertainty and risk. We establish central limit theorems for composite risk functionals. Furthermore, we discuss the asymptotic behavior of optimization problems whose objectives are composite risk functionals and we establish a central limit formula of their optimal values when an estimator of the risk functional is used. While the mathematical structures accommodate commonly used coherent measures of risk, they have more general character, which may be of independent interest.

Keywords Risk measures · Composite functionals · Central limit theorem

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