

# Improving the convergence rate and speed of Fisher-scoring algorithm: ridge and anti-ridge methods in structural equation modeling

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**Abstract** In structural equation modeling (SEM), parameter estimates are typically computed by the Fisher-scoring algorithm, which often has difficulty in obtaining converged solutions. Even for simulated data with a correctly specified model, non-converged replications have been repeatedly reported in the literature. In particular, in Monte Carlo studies it has been found that larger factor loadings or smaller error variances in a confirmatory factor model correspond to a higher rate of convergence. However, studies of a ridge method in SEM indicate that adding a diagonal matrix to the sample covariance matrix also increases the rate of convergence for the Fisher-scoring algorithm. This article addresses these two seemingly contradictory phenomena. Using statistical and numerical analyses, the article clarifies why both approaches increase the rate of convergence in SEM. Monte Carlo results confirm the analytical results. Recommendations are provided on how to increase both the speed and rate of convergence in parameter estimation.

**Keywords** Fisher-scoring algorithm · Coefficient of variation · Condition number · Speed of convergence

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