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A Bayes minimax result for spherically symmetric unimodal distributions

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Abstract We consider Bayesian estimation of the location parameter θ of a random vector X having a unimodal spherically symmetric density $f(\|x-\theta\|^2)$ for a spherically symmetric prior density $\pi(\|\theta\|^2)$. In particular, we consider minimaxity of the Bayes estimator $\delta_\pi(X)$ under quadratic loss. When the distribution belongs to the Berger class, we show that minimaxity of $\delta_\pi(X)$ is linked to the superharmonicity of a power of a marginal associated to a primitive of f. This leads to proper Bayes minimax estimators for certain densities $f(\|x-\theta\|^2)$.

Keywords Bayes estimators · minimax estimators · Spherically symmetric distributions · Location parameter · Unimodal densities · Quadratic loss · Superharmonic priors

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