

## U-statistics with conditional kernels for incomplete data models

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**Abstract** For incomplete data models, the classical U-statistic estimator of a functional parameter of the underlying distribution cannot be computed directly since the data are not fully observed. To estimate such a functional parameter, we propose a U-statistic using a substitution estimator of the conditional kernel given the observed data. This kernel estimator is obtained by substituting the non-parametric maximum likelihood estimator for the underlying distribution function in the expression of the conditional kernel. We study the asymptotic properties of the proposed U-statistic for several incomplete data models, and in a simulation study, we assess the finite sample performance of the Mann–Whitney U-statistic with conditional kernel in the current status model. The analysis of a real-world data set illustrates the application of the proposed methods in practice.

Keywords U-statistics  $\cdot$  Censored data  $\cdot$  Incomplete data models  $\cdot$  Non-parametric MLE

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