

On monotonicity of expected values of some run-related distributions

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Abstract We prove that the expectation of the binomial distribution of order k with success probability p is monotonically increasing with respect to p for all n and k . The result is extended to the problems on exchangeable random sequences and expectations of distributions of mixtures of binomial distributions of order k are studied. If the mixing measure is stochastically increasing with respect to its parameter, the expectation of the mixture of binomial distributions of order k becomes nondecreasing. As examples of mixing measures two submodels of beta distributions are examined and the resulting expectation of the mixture distribution is monotonically strictly increasing. Further, we prove some properties on the expectation of the ℓ -overlapping 1-runs in a sequence of independent and identically distributed n trials.

Keywords Binomial distribution of order k · Method of moments · Exchangeability · Negative hypergeometric distribution of order k · Beta distribution

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