

The complex multinormal distribution, quadratic forms in complex random vectors and an omnibus goodness-of-fit test for the complex normal distribution

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Received: 6 March 2014 / Published online: 21 September 2014
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Abstract This paper first reviews some basic properties of the (noncircular) complex multinormal distribution and presents a few characterizations of it. The distribution of linear combinations of complex normally distributed random vectors is then obtained, as well as the behavior of quadratic forms in complex multinormal random vectors. We look into the problem of estimating the complex parameters of the complex normal distribution and give their asymptotic distribution. We then propose a virtually omnibus goodness-of-fit test for the complex normal distribution with unknown parameters, based on the empirical characteristic function. Monte Carlo simulation results show that our test behaves well against various alternative distributions. The test is then applied to an fMRI data set and we show how it can be used to “validate” the usual hypothesis of normality of the outside-brain signal. An \mathbb{R} package that contains the functions to perform the test is available from the authors.

Keywords Characteristic function · Complex multinormal distribution · Complex normality test · Complex random vectors · fMRI · Goodness-of-fit test · Independence · Quadratic forms