

On a class of circulas: copulas for circular distributions

M. C. Jones · Arthur Pewsey · Shogo Kato

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Abstract This article is concerned with the analogue of copulas for circular distributions, which we call ‘circulas’. We concentrate on one particular class of circulas, which is pre-existing but not studied in such explicit form or detail before. This class is appealing in many ways but does not necessarily result in especially attractive bivariate circular models for arbitrary non-uniform marginals. A major exception to this is an elegant bivariate wrapped Cauchy distribution previously proposed and developed by two of the current authors. We look both at properties of the circulas themselves, including their density behaviour, distribution function, and dependence measures, and at properties of various distributions based on these circulas by transformation to non-uniform marginal distributions. We consider inference for the latter distributions and present two applications of them to modelling data. We concentrate mostly on the bivariate case, but also briefly consider extension to the multivariate case.

Keywords Bivariate · Circular distributions · Copula · Dependence · Multivariate · Torus