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Smooth change point estimation in regression models with random design

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Abstract We consider the problem of estimating the location of a change point θ_0 in a regression model. Most change point models studied so far were based on regression functions with a jump. However, we focus on regression functions, which are continuous at θ_0. The degree of smoothness q_0 has to be estimated as well. We investigate the consistency with increasing sample size n of the least squares estimates $(\hat{\theta}_n, \hat{q}_n)$ of (θ_0, q_0). It turns out that the rates of convergence of $\hat{\theta}_n$ depend on q_0: for q_0 greater than $1/2$ we have a rate of $\sqrt{n}$ and the asymptotic normality property; for q_0 less than $1/2$ the rate is $n^{1/(2q_0+1)}$ and the change point estimator converges to a maximizer of a Gaussian process; for q_0 equal to $1/2$ the rate is $\sqrt{n \cdot \ln(n)}$. Interestingly, in the last case the limiting distribution is also normal.

Keywords Regression · Change points · M-estimates · Rate of consistency · Asymptotic distribution