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Spacings around an order statistic

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Abstract We determine the joint limiting distribution of adjacent spacings around a central, intermediate, or an extreme order statistic $X_{k:n}$ of a random sample of size n from a continuous distribution F. For central and intermediate cases, normalized spacings in the left and right neighborhoods are asymptotically i.i.d. exponential random variables. The associated independent Poisson arrival processes are independent of $X_{k:n}$. For an extreme $X_{k:n}$, the asymptotic independence property of spacings fails for F in the domain of attraction of Fréchet and Weibull ($\alpha \neq 1$) distributions. This work also provides additional insight into the limiting distribution for the number of observations around $X_{k:n}$ for all three cases.

Keywords Spacings · Uniform distribution · Central order statistics · Intermediate order statistics · Extremes · Poisson process