Intrinsic means on the circle: uniqueness, locus and asymptotics

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Abstract This paper gives a comprehensive treatment of local uniqueness, asymptotics and numerics for intrinsic sample means on the circle. It turns out that local uniqueness as well as rates of convergence are governed by the distribution near the antipode. If the distribution is locally less than uniform there, we have local uniqueness and asymptotic normality with a square-root rate. With increased proximity to the uniform distribution the rate can be arbitrarily slow, and in the limit, local uniqueness is lost. Further, we give general distributional conditions, e.g., unimodality, that ensure global uniqueness. Along the way, we discover that sample means can occur only at the vertices of a regular polygon which allows to compute intrinsic sample means in linear time from sorted data. This algorithm is finally applied in a simulation study demonstrating the dependence of the convergence rates on the behavior of the density at the antipode.

Keywords Circular statistics · Directional statistics · Intrinsic mean · Central limit theorem · Asymptotic normality · Convergence rate