

Root n estimates of vectors of integrated density partial derivative functionals

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Abstract Based on a random sample of size n from an unknown d -dimensional density f , the nonparametric estimations of a single integrated density partial derivative functional as well as a vector of such functionals are considered. These single and vector functionals are important in a number of contexts. The purpose of this paper is to derive the information bounds for such estimations and propose estimates that are asymptotically optimal. The proposed estimates are constructed in the frequency domain using the sample characteristic function. For every d and sufficiently smooth f , it is shown that the proposed estimates are asymptotically normal, attain the optimal $O_p(n^{-1/2})$ convergence rate and achieve the (conjectured) information bounds. In simulation studies the superior performances of the proposed estimates are clearly demonstrated.

Keywords Bandwidth selection · Characteristic function · Convergence rate · Cross-validation · Multivariate kernel estimate · Nonparametric information bound