

Approximate tail probabilities of the maximum of a chi-square field on multi-dimensional lattice points and their applications to detection of loci interactions

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Abstract In this study, we define a chi-square random field on a multi-dimensional lattice points index set with a direct product covariance structure and consider the distribution of the maximum of this random field. We provide two approximate formulas for the upper tail probability of the distribution based on nonlinear renewal theory and an integral-geometric approach called the volume-of-tube method. This study is motivated by the detection problem of the interactive loci pairs which play an important role in forming biological species. The joint distribution of scan statistics for detecting the pairs is regarded as the chi-square random field above, and hence the multiplicity-adjusted p -value can be calculated using the proposed approximate formulas. By using these formulas, we examine the data of Mizuta, Harushima and Kurata (Proc Nat Acad Sci USA 107(47):20417–20422, 2010) who reported a new interactive loci pair of rice inter-subspecies.

Keywords Bateson–Dobzhansky–Muller model · Epistasis · Euler characteristic heuristic · Experimental crossing · Multiple testing · Nonlinear renewal theory · QTL analysis · Volume-of-tube method