

## Multicanonical MCMC for sampling rare events: an illustrative review

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**Abstract** Multicanonical MCMC (Multicanonical Markov Chain Monte Carlo; Multicanonical Monte Carlo) is discussed as a method of *rare event sampling*. Starting from a review of the generic framework of importance sampling, multicanonical MCMC is introduced, followed by applications in random matrices, random graphs, and chaotic dynamical systems. Replica exchange MCMC (also known as parallel tempering or Metropolis-coupled MCMC) is also explained as an alternative to multicanonical MCMC. In the last section, multicanonical MCMC is applied to data surrogation; a successful implementation in surrogating time series is shown. In the appendix, calculation of averages and normalizing constant in an exponential family, phase coexistence, simulated tempering, parallelization, and multivariate extensions are discussed.

**Keywords** Multicanonical MCMC · Wang–Landau algorithm · Replica exchange MCMC · Rare event sampling · Random matrix · Random graph · Chaotic dynamical system · Exact test · Surrogation