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Momentum-space approach to asymptotic expansion for stochastic filtering

Masaaki Fujii

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Abstract This paper develops an asymptotic expansion technique in momentum space for stochastic filtering. It is shown that Fourier transformation combined with a polynomial-function approximation of the nonlinear terms gives a closed recursive system of ordinary differential equations (ODEs) for the relevant conditional distribution. Thanks to the simplicity of the ODE system, higher-order calculation can be performed easily. Furthermore, solving ODEs sequentially with small sub-periods with updated initial conditions makes it possible to implement a *substepping method* for asymptotic expansion in a numerically efficient way. This is found to improve the performance significantly where otherwise the approximation fails badly. The method is expected to provide a useful tool for more realistic financial modeling with unobserved parameters and also for problems involving nonlinear measure-valued processes.

Keywords Zakai equation · Polynomial-function approximation · Measure-valued process