Ann Inst Stat Math (2014) 66:75–92
DOI 10.1007/s10463-013-0406-0

Bootstrapping continuous-time autoregressive processes

Peter J. Brockwell · Jens-Peter Kreiss · Tobias Niebuhr

Received: 4 October 2012 / Revised: 16 March 2013 / Published online: 9 May 2013

Abstract We develop a bootstrap procedure for Lévy-driven continuous-time autoregressive (CAR) processes observed at discrete regularly-spaced times. It is well known that a regularly sampled stationary Ornstein–Uhlenbeck process [i.e. a CAR(1) process] has a discrete-time autoregressive representation with i.i.d. noise. Based on this representation a simple bootstrap procedure can be found. Since regularly sampled CAR processes of higher order satisfy ARMA equations with uncorrelated (but in general dependent) noise, a more general bootstrap procedure is needed for such processes. We consider statistics depending on observations of the CAR process at the uniformly-spaced times, together with auxiliary observations on a finer grid, which give approximations to the derivatives of the continuous time process. This enables us to approximate the state-vector of the CAR process which is a vector-valued CAR(1) process, and whose sampled version, on the uniformly-spaced grid, is a multivariate AR(1) process with i.i.d. noise. This leads to a valid residual-based bootstrap which allows replication of CAR(p) processes on the underlying discrete time grid. We show that this approach is consistent for empirical autocovariances and autocorrelations.

Keywords CARMA processes · Lévy process · Bootstrap · Autocovariance