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On the convergence rate of the unscented transformation

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Abstract Nonlinear state-space models driven by differential equations have been widely used in science. Their statistical inference generally requires computing the mean and covariance matrix of some nonlinear function of the state variables, which can be done in several ways. For example, such computations may be approximately done by Monte Carlo, which is rather computationally expensive. Linear approximation by the first-order Taylor expansion is a fast alternative. However, the approximation error becomes non-negligible with strongly nonlinear functions. Unscented transformation was proposed to overcome these difficulties, but it lacks theoretical justification. In this paper, we derive some theoretical properties of the unscented transformation and contrast it with the method of linear approximation. Particularly, we derive the convergence rate of the unscented transformation.

Keywords Unscented transformation · Nonlinear transformation · Monte Carlo · Linear approximation