

## Alexander duality in experimental designs

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**Abstract** If  $F$  is a full factorial design and  $D$  is a fraction of  $F$ , then for a given monomial ordering, the algebraic method gives a saturated polynomial basis for  $D$  which can be used for regression. Consider now an algebraic basis for the complementary fraction of  $D$  in  $F$ , built under the same monomial ordering. We show that the basis for the complementary fraction is the Alexander dual of the first basis, constructed by shifting monomial exponents. For designs with two levels, the Alexander dual uses the traditional definition for simplicial complexes, while for designs with more than two levels, the dual is constructed with respect to the basis for the design  $F$ . This yields various new constructions for designs, where the basis and linear aberration can easily be read from the duality.

**Keywords** Alexander dual · Factorial design · Linear aberration