Ann Inst Stat Math (2012) 64:1227–1259
DOI 10.1007/s10463-012-0352-2

On the meaning of mean shape: manifold stability, locus and the two sample test

Stephan F. Huckemann

Received: 27 May 2011 / Revised: 26 September 2011 / Published online: 13 March 2012

Abstract Various concepts of mean shape previously unrelated in the literature are brought into relation. In particular, for non-manifolds, such as Kendall's 3D shape space, this paper answers the question, for which means one may apply a two-sample test. The answer is positive if intrinsic or Ziezold means are used. The underlying general result of manifold stability of a mean on a shape space, the quotient due to an proper and isometric action of a Lie group on a Riemannian manifold, blends the slice theorem from differential geometry with the statistics of shape. For 3D Procrustes means, however, a counterexample is given. To further elucidate on subtleties of means, for spheres and Kendall's shape spaces, a first-order relationship between intrinsic, residual/Procrustean and extrinsic/Ziezold means is derived stating that for high concentration the latter approximately divides the (generalized) geodesic segment between the former two by the ratio 1:3. This fact, consequences of coordinate choices for the power of tests and other details, e.g. that extrinsic Schoenberg means may increase dimension are discussed and illustrated by simulations and exemplary datasets.

Keywords Intrinsic mean · Extrinsic mean · Procrustes mean · Schoenberg mean · Ziezold mean · Shape spaces · Proper Lie group action · Slice theorem · Horizontal lift · Stratified spaces