Density-ratio matching under the Bregman divergence: a unified framework of density-ratio estimation

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Abstract Estimation of the ratio of probability densities has attracted a great deal of attention since it can be used for addressing various statistical paradigms. A naive approach to density-ratio approximation is to first estimate numerator and denominator densities separately and then take their ratio. However, this two-step approach does not perform well in practice, and methods for directly estimating density ratios without density estimation have been explored. In this paper, we first give a comprehensive review of existing density-ratio estimation methods and discuss their pros and cons. Then we propose a new framework of density-ratio estimation in which a density-ratio model is fitted to the true density-ratio under the Bregman divergence. Our new framework includes existing approaches as special cases, and is substantially more general. Finally, we develop a robust density-ratio estimation method under the power divergence, which is a novel instance in our framework.