On estimating distribution functions using Bernstein polynomials

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Abstract It is a known fact that some estimators of smooth distribution functions can outperform the empirical distribution function in terms of asymptotic (integrated) mean-squared error. In this paper, we show that this is also true of Bernstein polynomial estimators of distribution functions associated with densities that are supported on a closed interval. Specifically, we introduce a higher order expansion for the asymptotic (integrated) mean-squared error of Bernstein estimators of distribution functions and examine the relative deficiency of the empirical distribution function with respect to these estimators. Finally, we also establish the (pointwise) asymptotic normality of these estimators and show that they have highly advantageous boundary properties, including the absence of boundary bias.