Exponential inequalities and the law of the iterated logarithm in the unbounded forecasting game

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Abstract We study the law of the iterated logarithm in the framework of game-theoretic probability of Shafer and Vovk. We investigate hedges under which a game-theoretic version of the upper bound of the law of the iterated logarithm holds without any condition on Reality's moves in the unbounded forecasting game. We prove that in the unbounded forecasting game with an exponential hedge, Skeptic can force the upper bound of the law of the iterated logarithm without conditions on Reality's moves. We give two examples such a hedge. For proving these results we derive exponential inequalities in the game-theoretic framework which may be of independent interest. Finally, we give related results for measure-theoretic probability which improve the results of Liu and Watbled (Stochastic Processes and their Applications 119:3101–3132, 2009).