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Bayesian estimation of a covariance matrix with flexible prior specification

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Abstract Bayesian analysis for a covariance structure has been in use for decades. The commonly adopted Bayesian setup involves the conjugate inverse Wishart prior specification for the covariance matrix. Here we depart from this approach and adopt a novel prior specification by considering a multivariate normal prior for the elements of the matrix logarithm of the covariance structure. This specification allows for a richer class of prior distributions for the covariance structure with respect to strength of beliefs in prior location hyperparameters and the added ability to model potential correlation amongst the covariance structure. We provide three computational methods for calculating the posterior moment of the covariance matrix. The moments of interest are calculated based upon computational results via Importance sampling, Laplacian approximation and Markov Chain Monte Carlo/Metropolis–Hastings techniques. As a particular application of the proposed technique we investigate educational test score data from the project talent data set.

Keywords Gibbs sampling · Hierarchical analysis · Importance sampling · Laplacian approximation · Markov Chain Monte Carlo · Matrix logarithm transformation · Metropolis–Hastings algorithm · Volterra integral equation