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On the equivalence of the weighted least squares and the generalised least squares estimators, with applications to kernel smoothing

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Abstract This paper establishes the conditions under which the generalised least squares estimator of the regression parameters is equivalent to the weighted least squares estimator. The equivalence conditions have interesting applications in local polynomial regression and kernel smoothing. Specifically, they enable to derive the optimal kernel associated with a particular covariance structure of the measurement error, where optimality has to be intended in the Gauss-Markov sense. For local polynomial regression it is shown that there is a class of covariance structures, associated with non-invertible moving average processes of given orders which yield the Epanechnikov and the Henderson kernels as the optimal kernels.

Keywords Epanechnikov kernel · Local polynomial regression · Non-invertible moving average processes