Extended Bernstein prior via reinforced urn processes

Lorenzo Trippa · Paolo Bulla · Sonia Petrone

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Abstract A reinforced urn process, which induces a prior on the space of mixtures of Bernstein distributions is introduced. A nonparametric Bayesian model based on this prior is presented: the elicitation is treated and some connections with Dirichlet mixtures are given. In the last part of the article, an MCMC algorithm to compute the predictive distribution is discussed.

Keywords Bayesian nonparametrics · Bernstein polynomials · Polya urn schemes

1 Introduction

The Bernstein polynomial of degree \( k \) associated with a bounded function \( F \) on \([0, 1]\) is defined as

\[
B(x; k, F) = \sum_{j=0}^{k} F\left(\frac{j}{k}\right) \binom{k}{j} x^j (1-x)^{k-j}, \quad x \in [0, 1].
\]  

(1)

It is known that Bernstein polynomials well approximate \( F \) under general assumptions. If \( x \) is a continuity point of \( F \): \( \lim_{k \to \infty} B(x; k, F) = F(x) \). When \( F \) is a distribution function (df) the Bernstein approximation is also a df on \([0, 1]\) and,