

## Dual connections in nonparametric classical information geometry

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**Abstract** We construct an infinite-dimensional information manifold based on exponential Orlicz spaces without using the notion of exponential convergence. We then show that convex mixtures of probability densities lie on the same connected component of this manifold, and characterize the class of densities for which this mixture can be extended to an open segment containing the extreme points. For this class, we define an infinite-dimensional analogue of the mixture parallel transport and prove that it is dual to the exponential parallel transport with respect to the Fisher information. We also define  $\alpha$ -derivatives and prove that they are convex mixtures of the extremal ( $\pm 1$ )-derivatives.

**Keywords** Information geometry · Statistical manifold · Fisher metric · Orlicz spaces · Amari–Nagaoka duality