

## Frequentist and Bayesian measures of confidence via multiscale bootstrap for testing three regions

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**Abstract** A new computation method of frequentist  $p$  values and Bayesian posterior probabilities based on the bootstrap probability is discussed for the multivariate normal model with unknown expectation parameter vector. The null hypothesis is represented as an arbitrary-shaped region of the parameter vector. We introduce new functional forms for the scaling-law of bootstrap probability so that the multiscale bootstrap method, which was designed for a one-sided test, can also compute confidence measures of a two-sided test, extending applicability to a wider class of hypotheses. Parameter estimation for the scaling-law is improved by the two-step multiscale bootstrap and also by including higher order terms. Model selection is important not only as a motivating application of our method, but also as an essential ingredient in the method. A compromise between frequentist and Bayesian is attempted by showing that the Bayesian posterior probability with a noninformative prior is interpreted as a frequentist  $p$  value of “zero-sided” test.

**Keywords** Approximately unbiased tests · Bootstrap probability · Bias correction · Hypothesis testing · Model selection · Probability matching priors · Problem of regions · Scaling-law