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On V-orthogonal projectors associated with a semi-norm

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Abstract For any $n \times p$ matrix $\mathbf{X}$ and $n \times n$ nonnegative definite matrix $\mathbf{V}$, the matrix $\mathbf{X}(\mathbf{X}'\mathbf{V}\mathbf{X})^{+}\mathbf{X}'\mathbf{V}$ is called a $\mathbf{V}$-orthogonal projector with respect to the semi-norm $\|\cdot\|_{\mathbf{V}}$, where $(\cdot)^{+}$ denotes the Moore-Penrose inverse of a matrix. Various new properties of the $\mathbf{V}$-orthogonal projector were derived under the condition that $\mathrm{rank}(\mathbf{V}\mathbf{X}) = \mathrm{rank}(\mathbf{X})$, including its rank, complement, equivalent expressions, conditions for additive decomposability, equivalence conditions between two ($\mathbf{V}$-)orthogonal projectors, etc.

Keywords General linear model · Weighted least-squares estimator · $\mathbf{V}$-orthogonal projector · Moore-Penrose inverses of matrices · Rank formulas for partitioned matrix