Ann Inst Stat Math (2008) 60:901–917
DOI 10.1007/s10463-007-0124-6

# Semi-self-decomposable distributions on $\mathbf{Z}_+$

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Received: 19 August 2005 / Revised: 10 October 2006 / Published online: 7 March 2007
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**Abstract** We present a notion of semi-self-decomposability for distributions with support in $\mathbf{Z}_+$. We show that discrete semi-self-decomposable distributions are infinitely divisible and are characterized by the absolute monotonicity of a specific function. The class of discrete semi-self-decomposable distributions is shown to contain the discrete semistable distributions and the discrete geometric semistable distributions. We identify a proper subclass of semi-self-decomposable distributions that arise as weak limits of subsequences of binomially thinned sums of independent $\mathbf{Z}_+$-valued random variables. Multiple semi-self-decomposability on $\mathbf{Z}_+$ is also discussed.

**Keywords** Discrete distributions · Infinite divisibility · Semistability · Poisson mixtures · Probability generating functions · Weak convergence