Ann Inst Stat Math (2008) 60:883–900
DOI 10.1007/s10463-007-0139-z

Second-order nonlinear least squares estimation

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Received: 16 November 2005 / Revised: 14 November 2006 / Published online: 7 July 2007

Abstract The ordinary least squares estimation is based on minimization of the squared distance of the response variable to its conditional mean given the predictor variable. We extend this method by including in the criterion function the distance of the squared response variable to its second conditional moment. It is shown that this "second-order" least squares estimator is asymptotically more efficient than the ordinary least squares estimator if the third moment of the random error is nonzero, and both estimators have the same asymptotic covariance matrix if the error distribution is symmetric. Simulation studies show that the variance reduction of the new estimator can be as high as 50% for sample sizes lower than 100. As a by-product, the joint asymptotic covariance matrix of the ordinary least squares estimators for the regression parameter and for the random error variance is also derived, which is only available in the literature for very special cases, e.g. that random error has a normal distribution. The results apply to both linear and nonlinear regression models, where the random error distributions are not necessarily known.

Keywords Nonlinear regression · Asymmetric error distribution · Weighted least squares · Minimum distance estimator · Consistency · Asymptotic normality